**Monkey problems**

In a wood there stand 7 banana trees around an opening. There is a monkey who lives in these trees. An etholog is making scientific observations about the daily eating habit of the monkey. He visits the place every evening at 8 PM to find out which trees the monkey visited during the day.. He hopes to rely on footsteps in the sandy ground. He makes sketches of the footpaths on spot, and analyses them at home.
Can YOU tell where the monkey started, and where he is now on each of the days, according to the notes below?
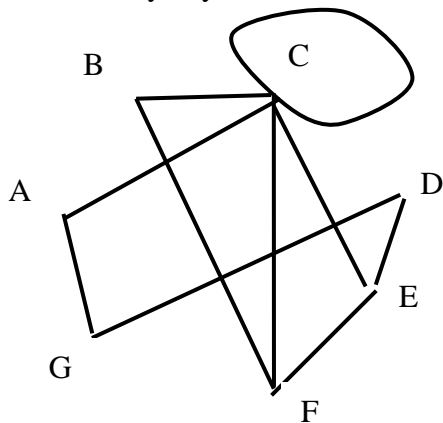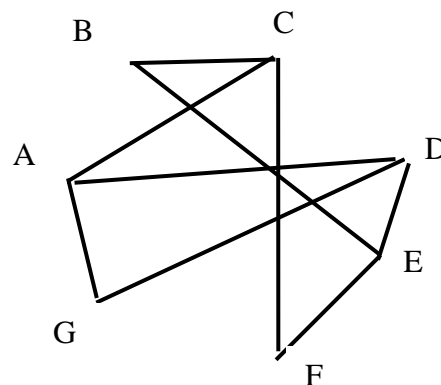
a)
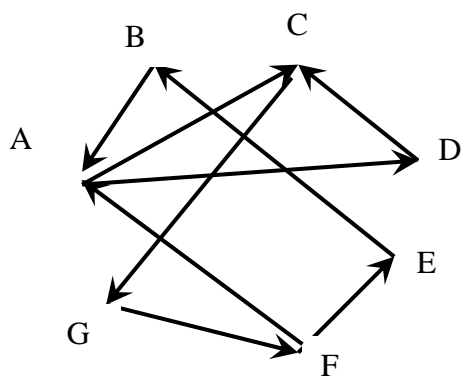
b) Rain washed away the footsteps such that the direction could not be seen.
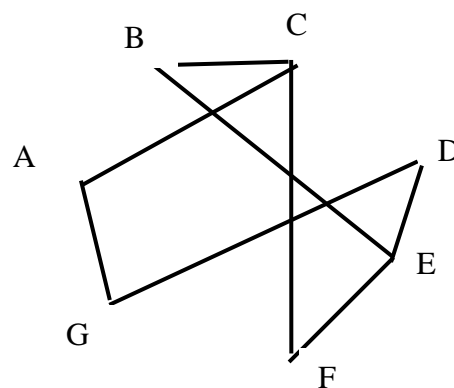
c) Another rainy day …

d) He forgot his notebook, so this is a drawing he made at home from his memory.

Traffic puzzles

1. In Brussels it is decided that 21 big cities o the EU must be well connected. They decided to have a direct flight between any two cities. How many flight services are there (direction does not matter, a service means flight connection between 2 cities)?

2. The direct flights scheme 1 of Brussels has been criticised as very uneconomical. So the transportation committee decided to have each city connected by direct flight with only 11 of other cities. However, up to now this plan could not be implemented . Can you help?

3. To be more economical, different proposals are considered to reduce the flights. Plan A wants to have the minimum number of direct flights such, that one can get to any city from any city by plain, albeit with changes. How many flights are needed at least?

4. Plan B is to utilise the existing high-speed train lines. So there are no flight connections where there is direct express train between the cities, otherwise yes. Somebody claims that the advantage of this plan is that all cities may be reached by plane, or by train only.

Party puzzles

1. Sherlock Holmes arrived at a party. There were already 8 people there. He asked everybody how many handshakes they made. He got the following numbers: 3, 0, 4, 1, 6, 5, 2, 6. He said: "Somebody did not tell the truth." How did he know this?

2. Sherlock Holmes claims that in any parties if you ask how many people one knows, there must be at least 2 people who have identical number of acquaintances. (Knowing each other is symmetrical, that is if A knows B, then B knows A too.)

3. In a party of 6, some people know each other, others do not. Show that there can be always 3 people found such that they mutually know each other, or none of them know each other.

    a. What can you say about a party of less than 6 people?
    b. What about a party of more than 6 people?

4. Mr and Mrs Jones invited 2 couples for dinner. After arriving, he asked everybody (including his wife) about the number of people they greeted by handshake. He got all different numbers. What number did his wife say?

5. * Same as 3, but for n guest couples.

Dominoes
1. You have a complete set of dominoes. How many pieces are there?
2. Can you make a complete closed circle by using all the dominoes?
3. How about if you take out all the doubles?

Some characteristics of graphs

1. In any *graph* the number of those vertices which are connected to odd number of vertices, is ….

2. Theorem of Euler (1736)

A graph has an Euler *circle* – that is a closed path consisting of all the edges onces - IF AND ONLY IF the following holds for the graph:

A graph has an Euler *path* – a sequence of all the edges, where the start and end point differs - IF AND ONLY IF the following holds for the graph:

3. A *tree* is a graph which is:
    - connected, that is there is a path between any two vertices;
    - does not contain circle (closed path).

   Decide how many edges does a tree have: A tree with n vertices has …………. edges.

4. A graph *is complete* if any two vertex is connected by an edge.

   A complete graph of n vertices has ……………. edges.